

# Accurate Prediction of Resistor Variation Using Minimum Sized Five-Resistor TLM

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## Abstract

In this work we show the importance of considering dimensional variation ( $\Delta L$  and  $\Delta W$ ) to accurately predict resistor variation for circuit design. To aid with our prediction, we first discuss the importance of measuring all physical components of a resistor ( $R_{SH}$ ,  $\Delta L$ , and  $\Delta W$ ) using the novel 5-R TLM technique and discuss advantages over conventional techniques. Then, we introduce and apply the propagation of errors statistical technique to formulate a method for accurately predicting resistor variation using nominal values and standard deviations of its components. Finally, we show how designers can exploit this statistical technique in predicting resistor variations in advance to save time and resources for a robust, high yield circuit design.

## INTRODUCTION

On-chip resistors are very useful in MMIC circuit design. They are used for current regulation, voltage division, impedance matching, gain and thermo-electric stabilization, and other requirements that ensure proper functioning of any circuit. Unfortunately, like other devices, on-chip resistors suffer manufacturing process variation and are never ideal. While resistive film thickness variation drives sheet resistance ( $R_{SH}$ ) variation, photolithography and etch process variations result in size variation which can significantly impact resistors drawn at or near the minimum size allowed. With the state of the art growth and deposition techniques, compound semiconductor foundries have obtained reasonably good control over the resistive film thickness but variations due to dimensions and their impact have either been ignored or the impact on circuit performance has not been well understood.

In this work, we first discuss the importance of extracting dimensional variation, especially of the width ( $\Delta W$ ). Secondly, we introduce a novel five-resistor transfer-length method (5-R TLM) and discuss the advantages in measuring all relevant resistor parameters over conventional methods. Finally, we discuss how to exploit “propagation of errors” [1] statistical analysis to more accurately predict variations for any given resistor type.

Fig. 1 shows a typical schematic of a rectangular resistor.  $L_D$  and  $W_D$  are drawn length and width of the resistor, while  $L_A$  and  $W_A$  are actual (electrical) dimensions as measured and extracted by various DC process control monitor (PCM) tests.  $\Delta L=L_A-L_D$  and  $\Delta W=W_A-W_D$  are the differences in the drawn vs. electrical dimensions. From drift theory and Ohm’s law, we know that any Ohmic resistor can be expressed as:

$$R = R_{SH} \frac{L_D + \Delta L}{W_D + \Delta W} + 2R_C \quad \text{----- (1)}$$

Where,  $R_C$  is the contact resistance of the resistor terminals.

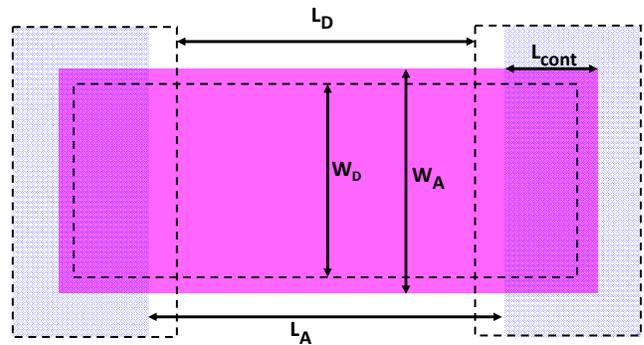


Figure 1- Typical definition of a resistor.

When drawing resistors, designers use standard resistors available in the process design kit (PDK) to make sure technology-relevant resistor and contact layers are used and that their designs adhere to the layer design rule. Most PDKs define library resistors with  $R_{SH}$  and  $R_C$  and may contain information about their standard deviations ( $\sigma$ ), but lack information about  $\Delta L$ ,  $\Delta W$  and their  $\sigma$ 's. While  $\Delta L$  can be approximated using  $R_C$  as  $\Delta L = 2R_C \times W/R_{SH}$ ,  $\Delta W$  needs a separate extraction. When circuit size is not a constraint, it is wise to draw resistors wide such that  $W_D \gg \Delta W$ . However, when the footprint is tight, it becomes necessary to draw resistors narrow and close to  $W_{min}$ . Thus, it becomes necessary to consider the impact of  $\Delta W$  on resistor variation.

To properly predict dimensional variation, we must have (a) accurately measured dimensional variation from utilizing an appropriate PCM structure and (b) a handy method for the

designer to predict resistor variation in advance using the measured component values. In relation to the above two requirements, the next two sections discuss a new 5-R TLM technique and the method's advantages in properly predicting resistor variation. The final section then shows how to use the propagation of errors technique to accurately predict resistor variation so that appropriate resistor selection is made for a robust, high-yield circuit design.

#### MEASURING DIMENSIONAL VARIATION

Most fabs use the van der Pauw (VDP) [2] or traditional wide TLM (Wide-TLM) structures to monitor  $R_{SH}$  (Fig. 2(a, b)). Wide-TLM structures also estimate  $\Delta L$  (Eq. 1), however, due to the lack of a width variable, neither VDP nor Wide-TLM methods can predict  $\Delta W$  and its variance. To solve this problem, a line width ( $L_{width}$ ) resistor (Fig. 2(c)) is typically used which takes in  $R_{SH}$  and  $L_D$  as input to estimate  $W_A$  and  $\Delta W$ . While the  $L_{width}$  structure is usually designed at  $W=W_{min}$ , Wide-TLMs typically have large sizes so

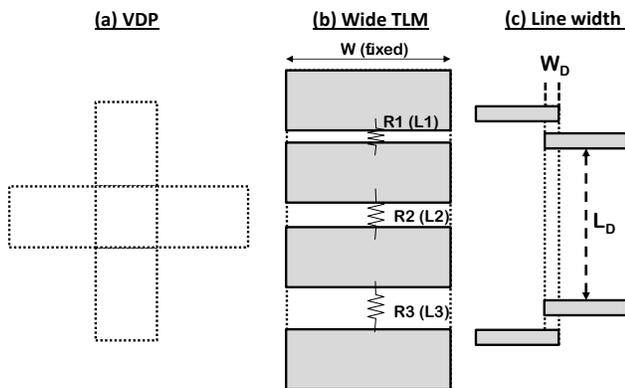


Figure 2- Conventional PCM Structures.

that the variations in  $R_{SH}$  is immune to variations in contact sizes and resistor dimensions. In practice, the resistors are not always drawn wide and the contact sizes can be comparable to resistor dimensions. Thus, for reliable extraction, resistors of all dimensions should be utilized. Also, due to the size and number of Kelvin contacts used, conventional structures tend to consume significant wafer footprint which could otherwise be used to fabricate additional useful die. To solve all of the above issues, we propose a five-resistor TLM (5-R TLM) structure with dimensions starting with nominal down to  $L_{min}$  and  $W_{min}$  (Fig. 3). Like Wide-TLMs, the 5-R TLM is designed with three different lengths ( $L_1$ =short ( $L_{min}$ ),  $L_2$ =intermediate,  $L_3$ =long) for  $R_{SH}$  and  $\Delta L$  extraction. Additionally, they also contain three different widths ( $W_1$ =wide,  $W_2$ =intermediate,  $W_3$ =narrow ( $W_{min}$ )) to help with the  $\Delta W$  extraction. Thus, one

is able to extract simultaneously  $R_{SH}$ ,  $\Delta L$  and  $\Delta W$  on the same PCM structure. An example strategy of length and width assignment to resistors R1-R5 is explained in Fig. 3(a). R1, with the shortest length, is expected to be most sensitive to  $\Delta L$  variation while R5 (narrow resistor) is expected to be most sensitive to  $\Delta W$  variation. One could argue that a three resistor TLM (3-R TLM) using a subset of resistors (R1, R2 and R5) of the 5-R TLM (Fig. 2(d)) could also suffice for  $\Delta L$  and  $\Delta W$  extraction. However, in the following section, we show that this 3-R TLM with only two width variant over-predicts  $\Delta W$  variance.

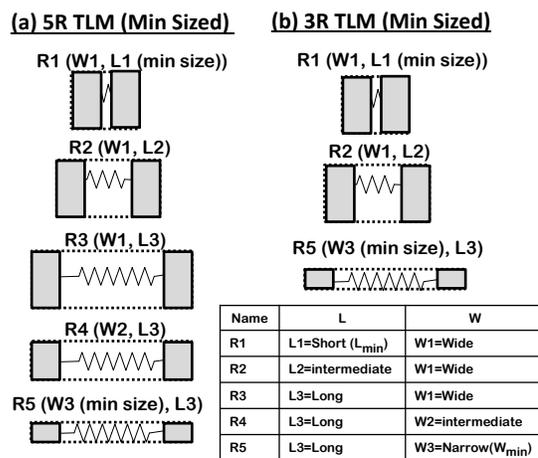


Figure 3- Proposed five-resistor TLM (5-R TLM) consisting of resistors R1-R5. Also, shown 3-R TLM (a subset) with resistors R1, R2 and R5.

Because the dimensions of the 5-R TLM resistors are small, probe pads can be arranged much closer thus occupying less space. Moreover, the five resistors can be Kelvin probed by sharing pads instead of using four separate connections for each resistor. To minimize pad count, the five resistors are first connected in series. When a resistor is having current forced across its two terminal pads, floating pads from other resistors can be used to sense the Kelvin voltage. This saves pad area and allows the TLM to be placed in a relatively tiny footprint.

#### COMPARISON OF METHODS

To compare the 5-R TLM against conventional methods, the 5-R TLM structure was included in standard Qorvo BIFET process [3]. Table I shows a summary of  $R_{SH}$ ,  $\Delta L$  and  $\Delta W$  distributions extracted from the thin film resistor (TFR) PCM test. From the summary, all methods measure similar mean ( $\mu$ ) values for  $R_{SH}$ ,  $\Delta L$  and  $\Delta W$ . However, the measured standard deviations are higher for the 5-R and the 3-R TLM methods.

This is expected as two out of five resistors are drawn at minimum allowed length (R1) or width (R5). Note, that the  $\sigma$  for the 3-R TLM method is the highest for all measured parameters. This is because the measurement is more prone to dispersion in R5 data with only two width variants. Fig. 4 illustrates this point by comparing  $\Delta W$  vs. R5 correlations between the 3-R vs. 5-R TLM methods.

TABLE I  
 $\mu$  AND  $\sigma$  OF ( $R_{SH}$ ,  $\Delta L$ ,  $\Delta W$ ) FOR DIFFERENT METHODS.  
Example: Thin Film Resistor (TFR)

Method	Rsh ( $\Omega/\square$ )			
	$\mu$	$\sigma$	N	$\sigma_M$
VDP/Lwidth	103.5	1.8	216	0.122
Wide TLM	102.4	1.8	216	0.122
3R (Min Sized)	102.9	5.3	216	0.361
5R (Min Sized)	103.2	2.2	216	0.150
Method	$\Delta L$ ( $\mu m$ )			
	$\mu$	$\sigma$	N	$\sigma_M$
VDP/Lwidth	-	-	-	-
Wide TLM	-0.1	0.21	216	0.014
3R (Min Sized)	-0.12	0.35	216	0.024
5R (Min Sized)	-0.14	0.25	216	0.017
Method	$\Delta W$ ( $\mu m$ )			
	$\mu$	$\sigma$	N	$\sigma_M$
VDP/Lwidth	0.59	0.037	216	0.002
Wide TLM	0.57	0.037	216	0.002
3R (Min Sized)	0.57	0.11	216	0.007
5R (Min Sized)	0.58	0.05	216	0.003

Clearly, for the same measured resistance, R5, the extracted  $\Delta W$  is noisier and the correlation is worse for the 3-R TLM. Thus, we discount using the 3-R TLM as a reliable measurement of variation.

#### VARIATION ANALYSIS FOR RESISTOR SELECTION

When estimating resistance from its physical parameters, PDK models typically provide nominal values for  $R_{SH}$  and then a combination of  $L_D$  and  $W_D$  is entered to obtain resistance. If space is a constraint, designers typically draw resistors at  $L=L_{min}$  and/or  $W=W_{min}$ . In this section, we show that, at these small or narrow dimensions, ignoring ( $\Delta L$ ,  $\sigma_{\Delta L}$ ) and ( $\Delta W$ ,  $\sigma_{\Delta W}$ ) can lead to erroneous predictions of resistor variation. For illustration, we focus on predicting resistance variation at  $W=W_{min}$  only, although similar analysis can also be conducted at  $L=L_{min}$ .

To accurately model variation, we use the “propagation of errors” method [1]. In statistical treatment of data, this method has been shown as a useful technique to predict variance of any quantity  $Q$  from the variances of its physical factors  $a, b, c, \dots$ . If  $Q = f(a, b, c, \dots)$ , and if we can independently

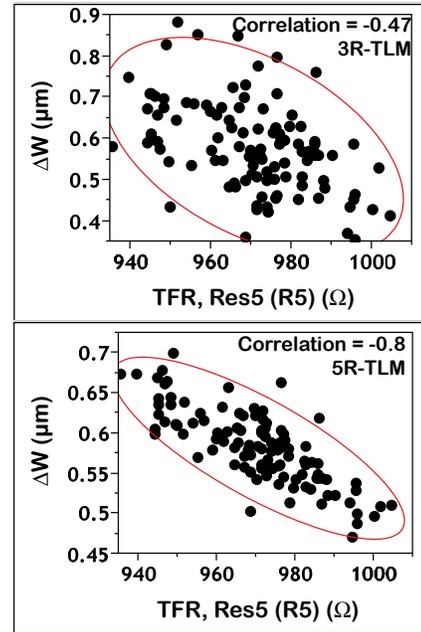


Figure 4- Comparison of  $\Delta W$  vs. R5 (Resistance) Correlation Between 3-R and 5-R TLM.

measure the factors  $a, b, c, \dots$ , then propagation of errors technique states that [1]

$$\left(\frac{\sigma_Q}{Q}\right)^2 = \left(\frac{1}{Q} \frac{\partial Q}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{1}{Q} \frac{\partial Q}{\partial b}\right)^2 \sigma_b^2 + \left(\frac{1}{Q} \frac{\partial Q}{\partial c}\right)^2 \sigma_c^2 + \dots \quad (2)$$

Since, resistance depends on  $R_{SH}$ ,  $\Delta L$  and  $\Delta W$ , we can predict variance of a rectangular resistor as follows:

$$R = R_{SH} \frac{L_D + \Delta L}{W_D + \Delta W} + 2R_C \quad (3)$$

$$\sigma_{R_{R_{SH}}} = \frac{\partial R}{\partial R_{SH}} \sigma_{R_{SH}} \quad (4)$$

$$\sigma_{R_{\Delta L}} = \frac{\partial R}{\partial (\Delta L)} \sigma_{\Delta L} \quad (5)$$

$$\sigma_{R_{\Delta W}} = \frac{\partial R}{\partial (\Delta W)} \sigma_{\Delta W} \quad (6)$$

Substituting  $Q = R$ ,  $a = R_{SH}$ ,  $b = \Delta L$  and  $c = \Delta W$  and solving Eqs. 2 - 6, one can show that

$$\frac{\sigma_R}{R} = \sqrt{\frac{\sigma_{R_{SH}}^2}{R_{SH}^2} + \frac{\sigma_{\Delta L}^2}{(L_D + \Delta L)^2} + \frac{\sigma_{\Delta W}^2}{(W_D + \Delta W)^2}} \quad (7)$$

Thus, for any desired resistor,  $\sigma_R/R$  can be predicted if nominal value ( $\mu$ ) and  $\sigma$  of  $R_{SH}$ ,  $\Delta L$  and  $\Delta W$  are known. To validate this model, Fig. 5 shows modeled (lines) vs. measured (symbol) variation ( $3\sigma_R/R$ ) for a thin film resistor. Clearly, all variation components,  $\sigma_{R_{SH}}$ ,  $\sigma_{\Delta L}$  and  $\sigma_{\Delta W}$  are nec-

essary to achieve a good fit. The model deviates if  $\Delta L$  is ignored at lower resistances and if  $\Delta W$  is ignored at higher resistances.

Fig. 6 shows the impact of varying width on  $\sigma_R/R$  vs.  $R$ . As the width is decreased to  $W_{min}$ , the knee of the variation shifts to higher and higher resistance. Thus, if a resistor is designed at  $W_{min}$ , one can use this method to predict the resistance (for a give resistor type) at which the variation starts to increase exponentially.

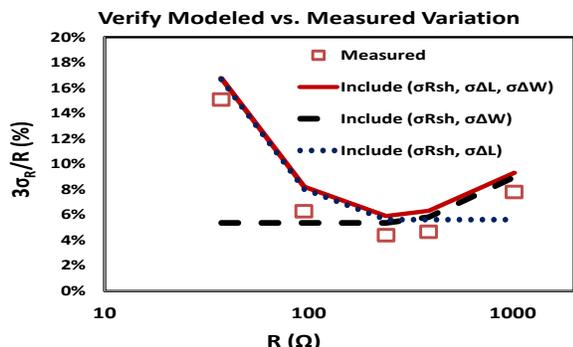


Figure 5 – Measured vs. Modeled  $3\sigma_R/R$  Variation as a Function of Component Variation.

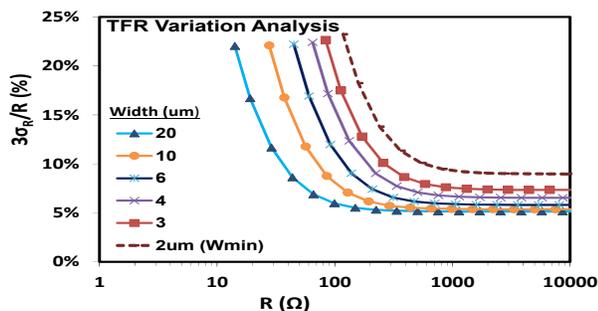


Figure 6 – Expected  $3\sigma_R/R$  Variation vs.  $R$  as a Function of Width.

To further illustrate the usefulness of this statistical treatment on resistors, we discuss the impact of ignoring  $\Delta W$  and  $\sigma_{\Delta W}$  on  $\sigma_R/R$  when a resistor of  $300\Omega$  is designed with a constraint of  $W=W_{min}$ . Fig. 7 shows predicted  $3\sigma_R/R$  for different Qorvo BIFET technology resistors with corresponding  $\mu$  and  $\sigma$  summarized in Table II. The three bars for each resistor type are separate solutions of Eq. 9 with component variations successively added starting with ( $R_{SH}$ ,  $\sigma_{RSH}$ ), then ( $\Delta L$ ,  $\sigma_{\Delta L}$ ) and finally ( $\Delta W$ ,  $\sigma_{\Delta W}$ ). If all the sources of variation are included and if the upper limit on acceptable variance is set at 20%, it would not be advisable to select emitter and TFR resistors. However, if  $\Delta W$  variation is ignored, the error in prediction becomes significant for emitter resistors. Unfortunately, such ignorance is typical and leads to more design spins, troubleshooting efforts, and unwanted delays in final circuit design.

In the worst case, if schedule is tight, wrong selection of resistor could lead to degraded die yield and higher cost per die.

TABLE II  
SUMMARY OF MEASURED DISTRIBUTION OF VARIOUS QORVO BIFET TECHNOLOGY RESISTORS WITH DESIGN RULE FOR MINIMUM SIZE AND SPACE

Parameter	TFR	Base	Emitter	Sub-collector	FET Contact Epi
Rsh ( $\Omega/\square$ )	103.2	271.60	33.56	9.23	105.90
dL ( $\mu m$ )	-0.14	0.64	0.41	8.58	1.76
dW ( $\mu m$ )	-0.58	0.84	-0.60	-1.09	-0.34
$\sigma_{Rsh}$ ( $\Omega/\square$ )	2.2	8.770	2.460	0.127	3.390
$\sigma_{dL}$ ( $\mu m$ )	0.25	0.087	0.420	1.118	0.199
$\sigma_{dW}$ ( $\mu m$ )	0.05	0.035	0.325	0.080	0.131
Min Size ( $\mu m$ )	2	4.9	2	4	3
Min Space ( $\mu m$ )	3	3	1.7	3	3

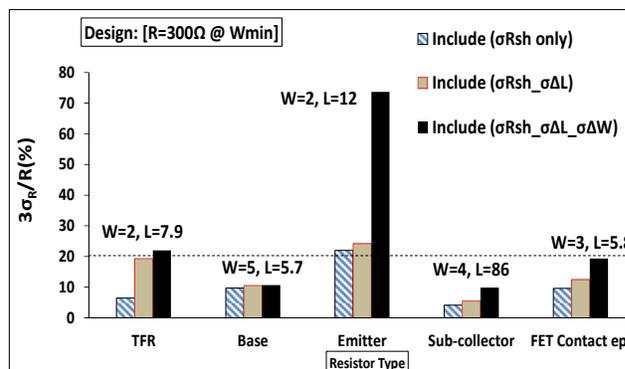


Figure 7 – Expected  $\% 3\sigma_R/R$  variation for a  $300\Omega$  Resistor Using Various Resistive Film Parameters from Qorvo BIFET technology.

## CONCLUSION

In summary, we discussed the importance of extracting dimensional variation ( $\Delta L$ ,  $\sigma_{\Delta L}$  and  $\Delta W$ ,  $\sigma_{\Delta W}$ ) of a resistor at minimum length and/ or width. Compared to conventional methods, we have shown that five-resistor TLM is a superior technique due to its capability to measure  $R_{SH}$ ,  $\Delta L$ ,  $\Delta W$  and their variations simultaneously. Using propagation of errors statistical technique, we have shown that variations for any type of fab resistor can be predicted using nominal values and  $\sigma$  of its components. Finally, we show the importance of including  $\Delta W$  and  $\sigma_{\Delta W}$  in resistor design.

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## ACRONYMS

- PCM: Process Control Monitor
- PDK: Process Design Kit
- TLM: Transfer Length Method
- VDP: van der Pauw